General Hyperological Principles [GHP]

Hyperological Community

June 1, 2022

The purpose of this document and the **GHP** framework is to articulate and concretize the methodology and background assumptions behind hyperology. If you are not already familiar with the overall concept of what hyperology is, please refer to the document *"Hyperology.pdf"*.

GHP doesn't rule out the mechanisms behind most hyperological fallacies, for the list of hyperological fallacies, please refer to the document *"HyperFallacies.pdf"*.

Metaprinciple

P0. Any hyperologically significant term used within principles of **GHP** such as "logic", "metaphysics" or "all" should not be interpreted by their hyperological meanings, and thus, principles of **GHP** cannot be hyperologically transcended.

(example: a hyperological structure defined to be "too vast for GM3 or GL2 to hold" is not metahyperologically-sound)

Principles of Mundanity

These principles concern basic hyperological stances on everyday life. [Fig. 1]

M1. Mundane Real Life [**MRL**] is the world we — human beings live in, it exists in some way and is a meaningful thing to consider and bring up in a conversation.

(Hyperology is incompatible with extreme forms of metaphysical, semantic or logical nihilism)

M2. **MRL** is a notion which needs no extra clarification beyond what is stated in **M1**.

(Exact nature of **MRL** is hyperologically insignificant)

M3. Mundane Assumptions [**MA**] are assumptions about **MRL** which are necessary for humans to believe or, at least, pragmatically assume, in order to function reasonably within **MRL**.

M4. **MA** are things which any hyperologist should assume while engaging in the activity of hyperology.

(It is never hyperologically productive to assume that any subset of **MA** is false. Hyperology is not a mysticist or anti-scientific practice)

Figure 1: Example of an event in **MRL**.

Principles of Anti-Formality

These principles concern the relation between hyperology and formal sciences. [Fig. 2]

AF1. There exists neither a *reasonable* nor a *meaningful* reduction of hyperology to an algorithmically-determined game of formal string manipulation.

(reasonable: feasible in practice;
meaningful: preserving intended meaning of terms)

AF2. There exists a reduction of all hyperology which will ever be performed by all hyperologists in **MRL** to an algorithmically-determined game of formal, finitary string manipulation. Such reductions of hyperology shall be notated as Hyperologically-Reductive Formalisms [**HRF**].

(as stated in **AF1**, **HRF**s are neither reasonable nor meaningful, yet their existence is nonetheless important to keep in mind)

AF3$_1$. Hyperology is not to be considered as a study of **HRF**s, instead, hyperologists should should assume[‡] the existence of hyperological concepts as if they are concrete, completed objects.

(It is never hyperologically productive to consider **HRF**s as intended semantics or logics behind hyperology. If you can't help but feel like hyperology is ultimately about **HRF**s, you most likely won't find hyperology meaningful or appealing)

AF3$_2$. Hyperologists should should assume[‡] the fact that the field of hyperology is the collection of all insights about those objects, regardless of whether this collection is realizable by a human individual or group.

(The most metaphysically "comfortable" stance on **HRF**s is that they could be indefinitely more complex, had **MRL** been an easier place to perform hyperology)

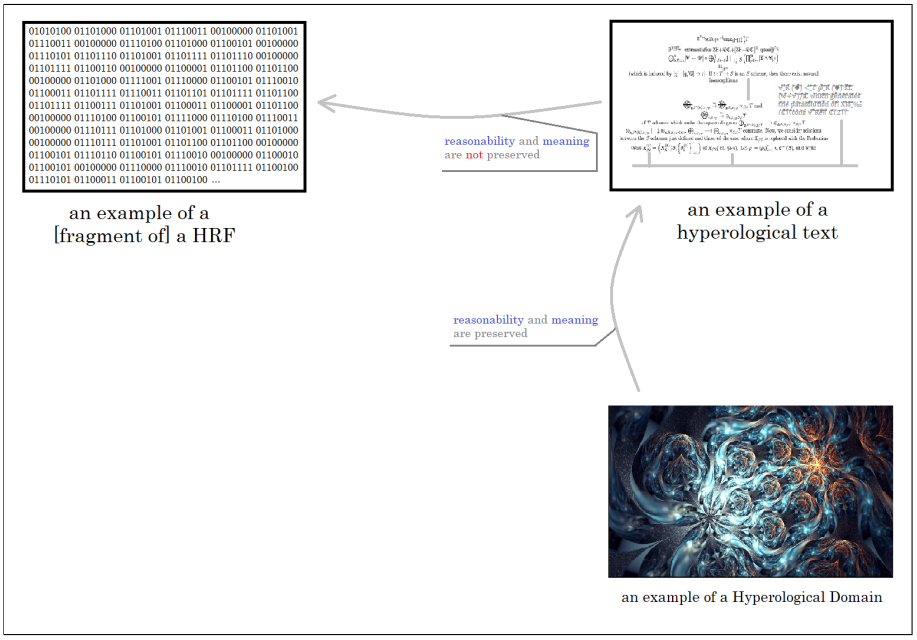

Figure 2: reduction of Hyperological Domains to **HRF**s.

Principles of Generality: Metaphysical

These principles concern necessary metaphysical assumptions for hyperology. [Fig. 3]

GM1. Hyperologists should should assume‡ the existence of the entire hyperological domain of discourse as a completed object which shall be notated as the $\forall^+$-Realm.

(This is not Box Absolutism in disguise, the purpose of **GM1** is not to set a general conceptual restriction on the domain of Hyperology, but to further aid Principles in this list which make the distinction between **MRL** and intended semantics of hyperology more simple and clear)

GM2. Hyperologists should assume‡ that the internal logic and metaphysics of the $\forall^+$-Realm are too complex and nuanced to be even vaguely defined at any point of hyperological practice.

(**GM2** prevents any attempts at general internal theory of the $\forall^+$-Realm from succeeding, forcing hyperologists to approach hyperology as an iterative and endless process of approximation the internal logic of the $\forall^+$-Realm. **GM2** is the meaning behind the prefix "hyper-" in hyperology [and its branches])

GM3. Hyperologists should treat the $\forall^+$-Realm as if it is completely and in all ways isolated from **MRL**.

(GM3 further prevents hyperological mysticism as well as sets the ultimate restriction on the semantics of such hyperological concepts as transfiction or thought-levels)

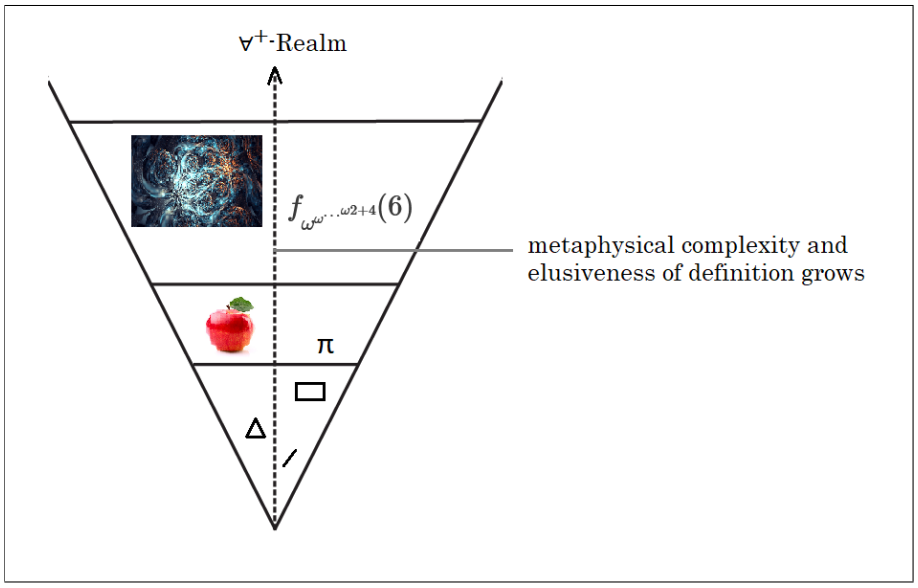

Figure 3: The overall picture of the $\forall^+$-Realm.

Principles of Generality: Logical

These principles concern logical and methodological assumptions for hyperology.

GL1$_1$. Hyperologists should should assume[‡] the existence of the informal framework of reasoning which shall be notated as Commonsense Reasoning [**CR**].

(**CR** acts as the informal analogue of the metatheory for hyperology)

GL1$_2$. **CR** contains no formal axioms or rules of inference and universally valid laws in general. The purpose of **CR** is not universal validity of arguments, but instead iterative clarification of arguments, until universal agreement and understanding of their *structure* and *nature* is achieved.

(structure of the argument: its map of deductive steps;
nature of the argument: its assumptions, background assumptions and inference rules)

GL2. Hyperological Proofs and Arguments should rely on the use of **CR** and **GHP**.

(hyperology is a fundamentally informal practice, which is nonetheless best carried out in a methodical and scientific manner)

Key:

‡ = (or, at least, suspend their disbelief in)

Important Terms:

MRL — Mundane Real Life [p.1]
MA — Mundane Assumptions [p.2]
reasonable reduction [p.3]
meaningful reduction [p.3]
HRF – Hyperologically-Reductive Formalisms [p.3]
$\forall^{+}$-Realm [p.5]
CR — Commonsense Reasoning [p.6]
structure of the argument [p.6]
nature of the argument [p.6]

The true magnitude of Absolute Totality

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Abstract

In this paper we introduce the reader to mental constructions which far surpass anything present in modern analytic philosophy and provide arguments against the well-established consensus that the scope of proper classes adequately reflects the magnitude of absolute Everything. An argument is provided that some of the greater mentally accessible constructions described in the paper are too great (in terms of cardinality and lengths of orderings) to have a formal structural ontology and so are incompatible with metaphysical positions such as ontic structural realism which some philosophers don't view as severely limiting.

1 Defence

To enter the realm of the constructions described in the paper we must first defend its conceptual coherence.

We shall present a nonstandard interpretation of the Burali-Forti paradox.

1.1 The modern perspective

As a consequence of the developments in the foundations of mathematics, set theory is seen as grounding mathematics as a whole. Formalists' conclusion is that all mathematical structures and phenomena can always be treated as sets with relations defined over them. Platonists' conclusion is that the realm of abstract mathematical objects is one-sorted — every abstract mathematical object is ontologically a set. Accordingly, proper-classes are seen as ethereal phenomena which do not bear actual existence as abstract individuals but instead lie at the very limit of the realm of abstract objects as a whole.

A major role in the formation of the view that proper classes as the untranscendible, or at least, not with philosophically fruitful results, was played by Georg Cantor's own views on the subject. Cantor linked the Absolute Infinite with God, and believed it satisfied the full reflection principle: every property of the Absolute Infinite is also held by some smaller object. Cantor's insight is so crucial that it still motivates the biggest subject in set theory — large cardinals. The stronger a large cardinal is the closer properties to full reflection principle it bears.

Burali-Forti paradox is then seen as reaffirming the conclusion that to speak of abstract objects is to speak of sets and that to speak of quantification over absolutely every abstract object (or even absolutely everything if one accepts ontic structural realism) is to quantify over the proper class V. Extending the idea, the consensus is that proper classes and higher-order collections are to be seen as nothing but byproducts of second-order and higher order quantification over V.

1.2 Burali-Forti paradox and its relatives

To present a different perspective on the Burali-Forti paradox we must first deconstruct it. Consider the following family of paradoxes:

FINITE WELL-ORDERINGS

Assume all well-orderings are finite (1)

Consider the well-ordering of all finite well-orderings, notate it as Ω (2)

Then, since all well-orderings are finite, Ω itself is a finite well-ordering and must be a proper initial segment of the well-ordering of all finite well-orderings

It follows that Ω is a proper initial segment of itself

For any finite well-ordering, its successor well-ordering is also finite

It follows that $\Omega + 1 < \Omega$ contradiction.

As a consequence, we are forced to abandon one of the two assumptions:

The assumption that all well-orderings are finite. (1)
The assumption that the well-ordering of all finite well-ordering exists. (2)

Set theorists, mathematicians and most modern analytical philosophers merrily abandon the assumption (1) and accept the realm of transfinite.

TYPES OF WELL-ORDERINGS IN GENERAL

Assume all well-orderings are computable/countable/less than 27th Woodin cardinal

Consider the well-ordering of all ——— well-orderings, notate it as Ω

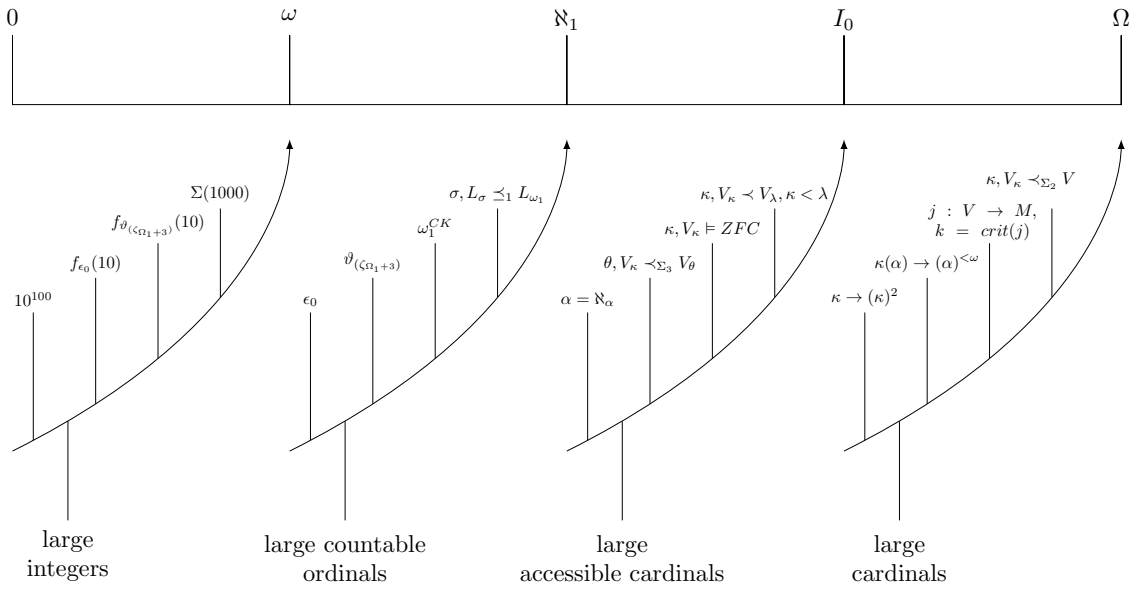
Then, since all well-orderings are ———, Ω itself is a ——— well-ordering and must be a proper initial segment of the well-ordering of all ——— well-orderings.

It follows that Ω is a proper initial segment of itself

For any ——— well-ordering, its successor well-ordering is also ———
It follows that $\Omega + 1 < \Omega$ contradiction.

As seen from this generalized argument the analogues of Burali-Forti paradox have exact same structure as the original paradox, and the analogues are valid as long as the supremum of all existing well-orderings is assumed to be a limit well-ordering. Set theorists have no issue breaking past this limit simply by considering more general kinds of well-orderings then the limit well-ordering admits.

However the original Burali-Forti paradox stands out as a barrier harder to break



Well-orderings can be grouped in several ontologically significant categories.

- Finite ordinals.
- Infinite ordinals. ω introduces the realm of transfinite structures with properties completely unseen in the realm of the finite. Ontological finitists already view infinite ordinals as nothing but a byproduct of mathematical "symbolic games".
- Uncomputable ordinals. Ordinals past ω_1^{CK} introduce intricacies of admissible ordinals unseen in the computable realm. Those who subscribe to metaphysical analogues of Church–Turing–Deutsch principle view uncomputable ordinals similarly.
- Uncountable ordinals. Ordinals past ω_1 introduce previously unseen uncountability and its consequences.
- Large cardinals. Ordinals past the first worldly cardinal introduce the vastly rich world of large cardinals.

However well-orderings past Ω introduce seemingly nothing. There is seemingly nothing special, no conceptual gain from considering $\Omega{+}1$ or $\Omega \cdot 2$ or Ω^+ except the technical and ontological difficulties behind such well-orderings. We are tempted to accept the conclusion that such well-orderings are not really ontologically grounded, are rather problematic while being completely unfruitful. After all, treating every abstract mathematical object ontologically as a set proves to be wildly successful and fruitful.

In fact, proper-class sized well-orderings are so similar to set-sized well-orderings that they are naturally elementarily equivalent to much smaller, large cardinal-sized well-orderings. Reductions of the considered well-orderings virtually never end up exceeding Mahlo cardinals. Those who considered the proper class -sized and greater well-orderings are then invited to study the large cardinal hierarchy, which is really where all of the fruitfulness lies. Suddenly, all hopes of finding something remarkable or ontologically meaningful about well-orderings past Ω are lost.

1.3 Platonists and Formalists

To find ontological meaning and non-triviality of proper class -sized and greater well-orderings we shall consider the following analogy:

Countable models $\mathcal{M}$ of ZFC fail to satisfy external fullness of the powerset of $\mathbb{N}$ which is something Platonists find philosophically unsatisfying about such models. Formalists, on the other hand, aren't

bothered by such models at all as the metatheoretically-definable subsets are the only kinds of subsets truly relevant to the formal study of set theory. As a consequence ω_1, to formalists, is nothing but a useful fiction and they have no need of ontological commitment to any ordinals greater than very large countable ordinals such that for all practical purposes they are indiscernible from uncountable ordinals.

Analogously, irreducibility of Ω to sets is viewed by set-theoretical Platonists as uncountability of ω_1 is seen by formalists - It doesn't bear ontological existence, since $(\Omega, <)$ is viewed by set theoretical Platonists not as an individual object but rather a byproduct of considering the realm of abstract objects in full generality. Set-theoretical Platonists, understandably, picture the realm of all abstract objects as proper-class sized, after all, the idea that every abstract mathematical object is a set is wildly successful. Extending this idea, Platonists picture absolute Everything as proper-class sized as other kinds of objects, beside abstract mathematical don't seem to significantly add up to the total amount. This position is ontologically comfortable for set-theoretical Platonists and is virtually never put to doubt by other philosophers. It is simply unlikely for them to seriously consider large cardinals let alone well-orderings past Ω in the setting of ontology in the first place, and even if they do, at the first glance Burali-Forti paradox seems to be an argument compelling enough against the impossibility of absolute Everything being any greater than Ω.

1.4 The alternative perspective

Ω can and should be appreciated as the supremum of all well-orderings amenable to a single logical sort, in case of Ω the sort involved is "set". Being the supremum, Ω is by an argument, obviously similar to Burali-Forti paradox, not amenable to a single logical sort. We shall notate the property of Ω of not being amenable to single logical sort as "1-sort irreducibility". [accordingly, being amenable to a single logical sort shall be notated as 1-sort reducibility].[‡]

Well-orderings α which are a β-sort irreducible shall be notated as $\alpha_{\dagger\beta}$.
The least well-ordering which is β-sort irreducible shall be notated as $\omega_{\dagger\beta}$.

1-sort irreducibility of $\omega_{\dagger 1}$ should be seen exactly in the same light as uncountability of ω_1. Just the same way the conception of ω_1 doesn't signify that we should stop and reaffirm that all mathematical objects are countable and that ω_1 bears no actual platonic existence, we should instead view the conception of 1-sort irreducibility of $\omega_{\dagger 1}$ as the first step towards something great, much greater in scale then the all of the previously considered conceptions. $\omega_{\dagger 1}$ instantly makes us wonder what kind of beasts $\omega_{\dagger 2}$ and $\omega_{\dagger\Omega}$ are, just like ω_1 makes us instantly wonder what kind of beasts ω_2 and ω_ω are. Usually many-sorted theories of sets and classes seem jumbled and uninspiring, however once we look at Ω from the perspective of 1-sort irreducibility we gain a newfound appreciation for such treatments.

1.5 Scales of α-sort irreducibility

Let us realize the massive scale of the leap from the supremum of all alpha-irreducible well-orders to the supremum of all alpha+1-irreducible well-orders.

α-sort irreducibility, unlike superclass-level isn't exhausted at the "cardinal successor", for lack of a better term, of a collection we started with. A well-ordering too great to be proper class-sized is a mere proper superclass and acts in about every way as "the next initial ordinal past Ω", while well-orderings too great to be 2-sort reducible are much *much* greater.

1.6 Standard models of a special type of Ω-sized theories

Consider a one-sorted theory $\mathfrak{T}$ with Ω-many constants c_α for each ordinal α and Ω-many axioms stating that the constants are pairwise non-equal. Let c_Ω be an additional constant stated to be pairwise non-equal to all c_α for ordinal α via Ω additional axioms. Assume ZFC_2 is a fragment of $\mathfrak{T}$. Let $\phi(x)$ be the predicate of being an initial ordinal. Let $\mathfrak{T}$ contain axioms of the form $\phi(c)$, for each

[‡]Further in the paper we shall present an argument why well-orderings of the form $\omega_{\dagger\beta}$ should be seen as β-th instances of the "full" reflection principle envisioned by Georg Cantor.

constant of the theory. Finally, let $\mathfrak{T}$ contain axioms of the form $\psi(c_\alpha, c_\Omega)$, for each ordinal α, where $\psi(\beta, \gamma)$ is a predicate "β is a lesser ordinal then γ".

Since $\mathfrak{T}$ is one-sorted it is sort-nonstandard, meaning all of its models are externally greater than than 1-sort reducible by size. If $\mathfrak{T}$ is viewed as object theory, it is externally, in other words, meta-theoretically unsound with respect to the amount of sorts in its models. If $\mathfrak{T}$ is viewed as metatheory, it is externally, in other words, meta-meta-theoretically unsound accordingly.

At cost of being sort-nonstandard, $\mathfrak{T}$ is able to axiomatize 1-sort irreducible collections greater in cardinality than Ω **concretely**, meaning without the circularity of using greater than Ω-many axioms or constants. There is no necessity to stop at $\mathfrak{T}$, one could easily construct theories with 2 cardinals above Ω, ω cardinals above Ω, a 53-huge cardinal above Ω, etc., modulo trivial adjustments to the construction of $\mathfrak{T}$.

However, the supremum of the size of least models of concrete theories is a clear limit. Let $\omega_{\dagger 1}^{\mathfrak{M}}$ be the notation of the supremum. It is notable that $\text{cof}(\omega_{\dagger 1}^{\mathfrak{M}}) = \Omega^+$. One could extend the construction to one-sorted theories which are not concrete or construct hierarchies of "concreteness degrees" akin to the hierarchy of admissible ordinals.

Such hierarchies get increasingly rich and layered, and thus reflection properties should eventually emerge. Initial segments α which satisfy such properties are the concreteness degree analogues of large cardinals and the limit of α which admit such reflection principles should be analogous to Ω. The property which grounds full reflection at Ω is its set-irreducibility, and analogously, the property which grounds full reflection at the limit is 2-sort irreducibility.

1.7 Nested limit-Universe construction

Consider a two-sorted theory of sets, one for "small" sets and one for "large" sets. Small sets are elements of $V_{<\kappa}$, κ being an instance of an inaccessible cardinal, and all sets are "large".

If axioms of ZFC hold for large sets and κ is the least inaccessible cardinal we get the simplest case of a doubly nested Grothendieck-Zermelo universe. Instead, we may have considered κ to be closed under taking the Mahlo operation or taking the next weakly compact, or the next 53-huge, or some larger cardinal each time building an increasingly larger initial Grothendieck-Zermelo universe, however, without any additional boost on how much further the top-layer Grothendieck-Zermelo universe extends. To extend the nested universe on both ends should have considered that the layer of large sets is also closed under taking Mahlo operation, weakly compact, 53-huge, etc., as long existence of proper-class amount of such cardinals is consistent. Extending our theory of a 2-nested Grothendieck-Zermelo universe on both layers with large cardinals we get increasingly rich structure.

Completing the idea, we may, philosophically, consider the limit of such extensions.* We would get a 2-nested Grothendieck-Zermelo universe whose bottom layer is already Ω-tall and whose outer layer extends to something much greater than mere next inaccessible, Mahlo or the next 53-huge. The height of the second layer would be $\omega_{\dagger 1}$ — the analogue of Ω for 1-sort irreducible collections. It would be the supremum of all 2-sort reducible and, as a consequence, be the least 2-sort irreducible well-ordering.

If we considered the analogue of the limit-extension of a 3-nested Grothendieck-Zermelo universe we would get $\omega_{\dagger 2}$ — the analogue of Ω for 2-sort reducible collections as the height of the overall intended interpretation.

We may then consider the similar limit extension for w-nested Grothendieck-Zermelo universes, I_0-nested Grothendieck-Zermelo universe and Ω-nested Grothendieck-Zermelo universes. We may also consider more intricate structures such as limit extensions of Ω+1-nested Grothendieck-Zermelo universes or limit-extensions of κ-nested Grothendieck-Zermelo universes, where κ is the least 2-sort irreducible well-ordering. If we wish the height of the intended interpretation to be the "12th extendible cardinal past the supremum of 2-sort reducible well-orders" we could cook up the appropriate limit-extension construction. In this particular example case we would start with a 3-sorted theory of a 3-nested Grothendieck-Zermelo universe. Postulate the full reflection of the small and moderate universes and postulate the existence of sets up to the next 12-extendible on the top layer.

*Mere closure under large cardinals is far, far from enough due to concreteness degrees, yet it is still possible to mentally address such proper completion in the same manner Platonists mentally address Th($\mathbb{N}$) or (Ω, $<$).

In fact, there is no need to stop at α-sort irreducibility for finite α. We can conceptualize and appreciate the colossal well-orders which are ω-sort irreducible, Ω-sort irreducible, $\varsigma = \varsigma_{\dagger\varsigma}$ etc.

1.8 Relation between $\mathcal{P}(\alpha)$ and the α-sort successor operation

It may seem straightforward that the collection of all α-sort irreducible well-orderings is closed under the powercollection operation, for each well-ordering α, however, without explicitly assuming this principle as an axiom it is consistent that it is not the case. Under the perspective of α-sort irreducibility the axiom of powerset in ZFC, states that powerset operation preserves 1-reducibility, we shall propose a similar, more general axiom.

Global Powercollection †-Idempotence. For all well-orderings α,

$$\dagger(\mathcal{P}(\alpha)) = \dagger(\alpha)$$

Without the presence of **GPI**, well-orderings α such that $\dagger(\mathcal{P}(\alpha)) = \dagger(\alpha)$ shall be notated as †-idempotent.

A slightly weaker axiom shall also be considered.

Moderate Powercollection †-Idempotence. For all well-orderings α, β

$$(\dagger(\mathcal{P}(\alpha)) = \dagger(\alpha)) \longrightarrow ((\dagger(\beta) = \dagger(\alpha)) \longleftrightarrow (\dagger(\mathcal{P}(\beta)) = \dagger(\alpha))$$

While **GPI** might be false any set theoretical platonist would agree that **MPI** holds at least in some structures.

Axioms such as **GPI** and **MPI** may or may not hold in initial segments of the hierarchy of all α-sort irreducible well-orderings ordered under order-preserving embeddings. **GPI** certainly holds in V and its violation in $\omega_{\dagger 1}$ would contradict the principle of *minimal cumulative structural violation.*

Under the same reasoning, it should be the case that $\omega_{\dagger 2}$, $\omega_{\dagger 3}$, etc. $\models$ **GPI**. On the other hand violation of **GPI** is less structurally severe than violation of well-orderability so the least well-ordering α such that $\alpha \not\models$ **GPI** must exist and should in some way contradict the premises of the proof above. This observation hints towards the hierarchy of instances of violations of **GPI** and principles similar to it. Perhaps some kinds of such instances play the role of "large cardinal" properties in the realm of α-sort irreducble well-orderings.

1.9 Large Cardinals beyond Ω

The notion of α-sort irreducibility provides us with a completely new perspective on the well-orderings past Ω and allows us to create a hierarchy of increasing scope and complexity akin to the hierarchy of large cardinals or large countable ordinals.

Definition 1.1 (α-sort fixed point)**.** κ is α-sort-inaccessible iff κ is a κ-sort irreducible

Definition 1.2 (α-sort inaccessible)**.** κ is α-sort-inaccessible iff κ is a κ-sort irreducible and $cof(\kappa) = \kappa$

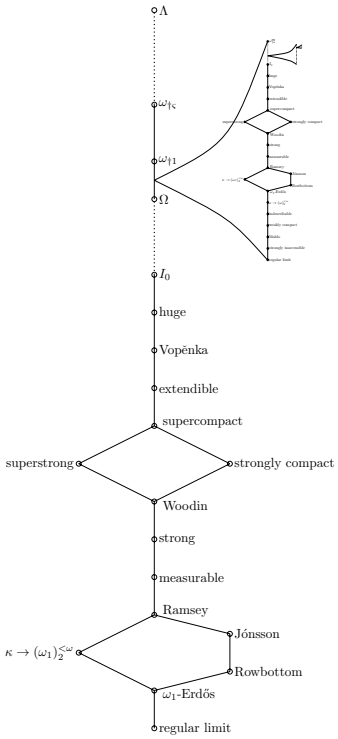
Definition 1.3 (α-sort Mahlo)**.** κ is α-sort Mahlo iff κ is κ-sort irreducible, $cof(\kappa) = \kappa$ any normal function f : $\kappa \to \kappa$ has a α-sort inaccessible fixed point.

Most variations of fixed points, inaccessibles and Mahlo cardinals translate straightforwardly into α-sort irreducible analogues.

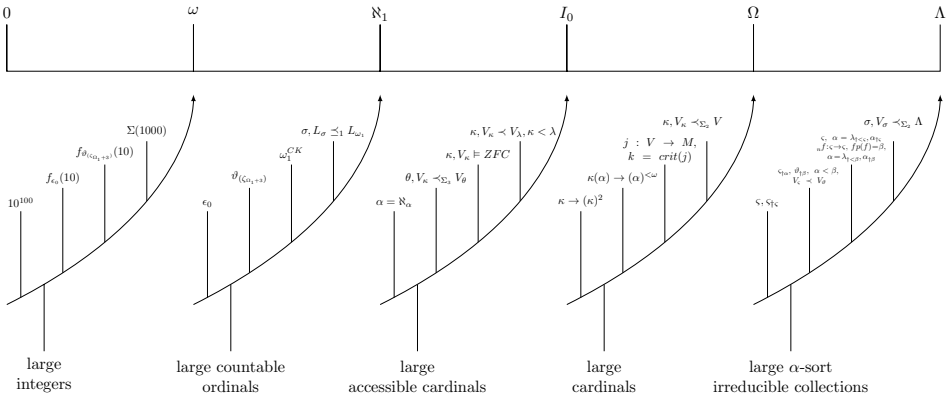
1.10 Nested Large Cardinal hierarchies

Hierarchies of increasingly reflective initial segments of the hierarchy of all well-orderings become increasingly deeply nested in rapid succession. Figuratively speaking, the hierarchy of concreteness degrees relates to the hierarchy of large α-sort irreducible well-orderings the same way the hierarchy of recursively large ordinals relates to the hierarchy of large cardinals.

$$\text{rec. large } \kappa \xmapsto{\omega_1 \mapsto \Omega} \text{large } \kappa \;\cong\; \omega^{\mathfrak{M}}_{\dagger\alpha} \xmapsto{\omega_{\dagger 1} \mapsto \Lambda} \text{large } \omega_{\dagger\alpha}$$



Recognizing ω_1 as "recursively Ord" and $\omega_{\dagger 1}$ as "concretely Λ" we can now fully envision the mathematical non-trivially in well-orderings past Ω. It is now particularly clear that, while Ω contains only a single copy of the large cardinal hierarchy, most well-orderings past Ω contain colossal amounts of similar such hierarchies simultaneously. As a consequence, one may study their ordering, consider special kinds of well-orderings which form initial segments of particular hierarchies or create and study more intricate notions related to the hierarchies. It is unclear how deeply nested the hierarchies can get and establishing the approximate limit is a also possible direction for future research.



2 Going much further

2.1 Deconstructing the concept of well-orderings

Having made a huge leap into the realm of α-sort irreducible collections we may still go forward and make a colossal leap.

What if we consider the limit of all these α-sort irreducible well-orderings? The limit, in Burali-Forti fashion, would be something which is the supremum of the kind of well-orderings we are considering.

It might be not initially clear that α-sort irreducibility was our last resort for extending the concept of well-ordering past Ω. However it will be clear if we call to mind the definition of a well-order as a concept.

A well-order is precisely a well-founded linear order.

Note that the cardinality or size issues of the well-ordering do not cause contradictions as long as the sizes are well-orderable. For this reason a well-ordering of an amorphous set is inherently contradictory but well-ordering of a proper class or a greater collection is not.

Now, note that the concept of well-order κ which is α-sort irreducible is never inherently contradictory as long as κ and α are both well-orderable. If α is limit well-ordering then it is naturally cofinal in κ, otherwise it is embeddable as a well-order in κ (consider the initial well-orders which are $<\alpha$-sort irreducible, these initial segments of κ would be elements of the well-ordering of α under the same well-ordering inherited from κ). For this reason, once we run out of α-sort irreducible well-orderings we run out of well-orderings in general, regardless of their size.

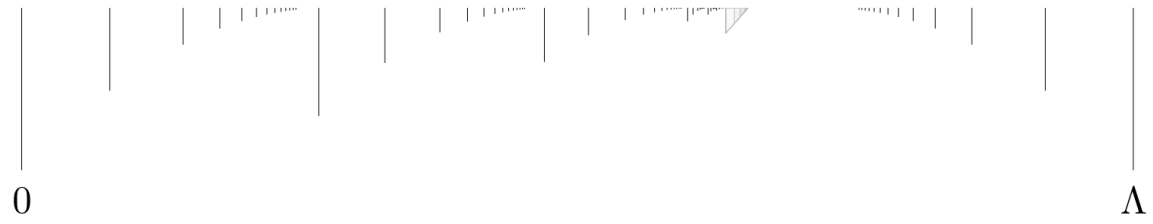
The only way to extend the hierarchy is to cast aside some of the essential non size-related properties of well-orderings. To compromise linearity of the ordering is clearly too drastic of a difference, our hierarchy was in many ways about length, so it is vital to the hierarchy. To compromise classical logic for paraconsistent is more reasonable but still is too hasty. We should compromise well-orderability minimally. Clearly, reconsidering the whole background logic may not be the most minimal compromise yet.

We shall turn our attention to well-foundedness, we shall consider a particular family properties which violate it. The properties form a very natural hierarchy by severity of the violation.[‡]

2.2 ω-almost-wellorderings

Define an order to be ω-almost-wellordered iff it is linear and contains no strictly decreasing chains of length ω^2, the motivation for such a definition will be clear further down the text.

In fact, for things to remain consistent and relatively similar to previous endeavours we need to drop another important property of well-orders. Supremum can no can longer always be the union of previous orders, as existence of such unions in some special cases is inconsistent. To work around that we declare the union to be slightly longer, in a precise sense, and restrict ourselves from taking the initial segment at exactly the union as a valid sub-collection.



Definition 2.1 (Λ)**.** Λ is the supremum of all well-orderings regardless of their collection types.

Λ, has no well-ordered initial segment that is equal to exactly the union of all well-orderings. Λ also contains "technical elements" at the end of it ordered as reverse ω. This is to cause slight ill-foundedness to avoid Burali-Forti paradox. A notable property of Λ is that any its initial segment is either isomorphic to itself or is well-ordered. Note that any decreasing sequence in Λ is not longer than $\omega + n$, for some finite ordinal n, so Λ is ω-almost-wellordered We can perform usual ordinal operations on Λ while retaining the essential properties of ordinal arithmetic as well as retaining ω-almost-wellorderability. Ordinal successor needs to be slightly redefined.

Definition 2.2 (The 3 fundamental types of ω-almost-wellorderings)**.**

1. Successor: λ which satisfy $\alpha + 1 = \lambda$ for some ω-almost-wellordering α.
2. Limit: λ such that $\forall \alpha < \lambda$, $\exists \beta$ such that $\alpha < \beta < \lambda$, α, β are ω-almost-wellorderings.
3. ω^*-point (special case of limit): λ such that $\exists$ technical b $\in \lambda$ not bounded by any non-technical a $\in \lambda$.

Definition 2.3 (Ordinal successor of an ω-almost-wellordering)**.** Ordinal successor of λ is the shortest (up to isomorphism) ω-almost-wellordered order extending λ not order isomorphic to λ.

[‡]In a precise sense, the property of self-containment is the supremum of the hierarchy.

Remark. *The definition prevents reverse-ω-points from being their own successors and prevents their successors from being limit.*

Definition 2.4 (Ordinal sum $\alpha + \beta$ of ω-almost-wellorderings and well-orderings)**.** Ordinal sum for non-ω^*-point α and arbitrary β is the lexicographic union $\alpha \cup \beta$; for ω^*-point α and arbitrary β is the lexicographic union $\alpha \cup \omega \cup \beta$.

Remark. *This definition prevent an analogous pathology.*

Definition 2.5 (Ordinal product $\alpha \cdot \beta$ of ω-almost-wellorderings and well-orderings)**.** Ordinal product of α and β is the lexicographic product $\alpha \times \beta$.

Definition 2.6 (Ordinal exponentiation α^β of ω-almost-wellorderings and well-orderings)**.** Ordinal exponentiation of α and β is the lexicographic order on the Cartesian power α^β.

Definition 2.7 (Principle of preservation of technical elements)**.** Technical elements are defined to be idempotent under left multiplication and left exponentiation for all ω-almost-wellorderings $\geq \Lambda$.

Remark. *This is to prevent pathological behavior of the operations at technical elements and make the hierarchy of ω-almost-wellorderings a more direct generalization of the hierarchy of ordinals.*

Theorem 1. *Λ+1 and Λ+α, where α is any well-order is ω-almost-wellordered; $\Lambda \cdot 2$ is ω-almost-wellordered.*

Proof. Throughout the orders at most 2 copies of ω^* are present, the rest of the order is well-ordered, meaning maximally long decreasing chains are of length $\omega \cdot 2 + n$, for finite ordinals n. □

Theorem 2. *$\Lambda \cdot \alpha$, where α is any well-order is ω-almost-wellordered.*

Proof. Throughout the order the copies of ω^* are ordered as α, which implies that, in the decreasing chains of the resulting order, elements of only finitely many copies of ω^* can be present [otherwise well-orderability of the order of copies of ω^* would be contradicted]. □

Theorem 3. *$\Lambda \cdot \lambda$ is an ω^*-point iff λ is any successor or ω^*-point.*

Proof. If λ is a successor then $\Lambda \cdot \lambda$ ends in a copy of λ which ends with technical elements ordered as reverse ω, If λ is an ω^*-point then $\Lambda \cdot \lambda$ by preservation of technical elements $\Lambda \cdot \lambda$ ends with technical elements ordered as reverse ω. If λ is limit then the resulting order contains neither the last copy of Λ nor preserved technical elements and hence, is a non ω^*-point limit. □

Theorem 4. *Λ^α, under ordinal exponentiation, where α is an arbitrarily well-order is ω-almost-wellordered.*

Proof. Analogous. □

Theorem 5. *Λ^λ, under ordinal exponentiation, is a reverse-ω-point iff λ is any successor or ω^*-point.*

Proof. Iff λ is successor Λ^λ is equal to $\Lambda^\alpha \cdot \Lambda$ for some α, and by preservation of technical elements the order ends with technical elements ordered as ω^*. Iff λ is an ω^*-point then by preservation of technical elements Λ^λ ends with technical elements ordered as ω^*. Iff λ is a non-ω^*-point limit then Λ^λ is the union $\Lambda^{\alpha<\lambda}$ which contains no greatest power of Λ and hence neither contains a greatest element nor ends with technical elements ordered as ω^*.

□

Definition of cardinal successor needs to be slightly adjusted as well.

Definition 2.8 (Cardinal successor of ω-almost-wellordering)**.** Cardinal successor of κ where κ is an ω-almost-wellordering is the least ω-almost-wellordering λ (up to isomorphism) such that there is no injection f : $\lambda \to \kappa$.

With this definition nothing prevents us from iterating cardinal successor over ω-almost-wellorderings.

Cofinality of ω-almost-wellorderings is especially well-behaved. If we exclude the impact of "technical elements" on lengths of cofinal sub-orders, cofinalities would behave exactly as expected.

Definition 2.9 (Cofinality of ω-almost-wellorderings)**.** $cof(\lambda)$ for ω-almost-wellorderings λ is defined as the least cardinality of sub-collections A $\subset \lambda$ such that there is no non-technical elements b $\in \lambda$ greater than all non-technical elements a $\in$ A

Theorem 6. *Cofinality of cardinal successor of Λ (Λ^+ for short) is Λ^+.*

Proof. Consider cof(Λ^+) $< \Lambda^+$, then Λ^+ is at most a union of Λ-many initial segments, taken at elements of the cofinal sequence, each sized at most Λ. This establishes a bijection between Λ^+ and at most $\Lambda \cdot 2$, however $\Lambda \cdot 2$ is equinumerous to Λ, contradiction. □

Theorem 7. *If κ is a cardinal successor of an ω-almost-wellordering λ then cof(κ) = κ.*

Proof. Analogous. □

[We immediately infer that if cof(κ) $< \kappa$, then κ is not a successor of any λ, in other words, the familiar property of successor and singular cardinals is preserved]

We may consider inaccessible ω-almost-wellorderings, Mahlo ω-almost-wellorderings, etc. Nothing prevents us from iterating even the "α-sort successor" operation on ω-almost-wellorderings. It behaves exactly the same way it does on well-orders. It doesn't interfere with any of the structural properties of orders involved as all it does is introduce an ontological shift to a higher type of collections, each iteration of α-sort successor acts as jump to a much muc much larger regular limit ω-almost-wellordering. Going through Λ, $\omega_{\dagger\Lambda+1}$, $\omega_{\dagger\Lambda+2}$, $\omega_{\dagger\Lambda^+}$, etc... We eventually get stuck once again and reach a special Burali-Forti limit.

Definition 2.10 (($\omega^* \cdot 2$)-limit)**.** An ω-almost-wellordering λ is ($\omega^* \cdot 2$)-limit iff λ contains an end segment of technical elements of order type $\omega^* \cdot 2$.

Although, it is consistent to assume that no ω-almost-wellordering λ is ($\omega^* \cdot 2$)-limit, such property of λ is less structurally violating than violation of ω-almost-wellorderability, so, due to the principle of *minimal cumulative structural violation* it is better to assume the existence of ω-almost-wellordered ($\omega^* \cdot 2$)-limits.

Remark. *Analogously defined ω-almost-wellordered $(\omega^* \cdot n)$-limits, for all finite ordinals n also exist under similar reasoning.*

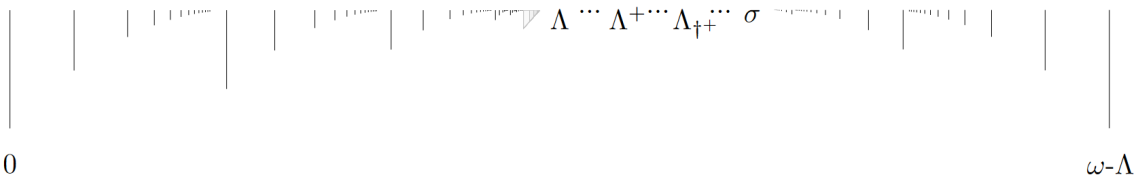
It is notable that the supremum of least $(\omega^* \cdot n)$-limits, for finite ordinals n, need not fail to be ω-almost-wellorderable. In fact, the supremum may not even be an ω^*-point, and, under axioms which cohere with the principle of *minimal cumulative structural violation*, it shouldn't be. It is also possible to consider $(\omega^* \cdot n)$-limits with large cardinal or reflective properties which, under the same principle, should be much larger than their non-$(\omega^* \cdot n)$-limit counterparts.

Finally, once more, we have to drop a structural property of the orders involved in our hierarchy. Dropping liearity of orders or consistency of the background logic once again proves to be too drastic of a change and we instead let go of well-orderability just slightly more drastically than before. We let go of ω-almost-wellorderability, and instead require our orders to be just ω^2-almost-wellordered.

2.3 ω^2-almost-wellorderings

Define an order to be ω^2-almost-wellordered iff it is linear and contains no strictly decreasing chains of length ω^3

Motivation for such a definition is now more clear. The hierarchy continues smoothly and is essentially the same as before



Definition 2.11 (Λ)**.** ω-Λ is the supremum of all ω-almost-wellorderings regardless of their collection types.

In similar spirit to Λ, ω-Λ has no ω-almost-wellordered initial segment that is equal to exactly the union of all ω-almost-wellorderings. Technical elements at the end of ω-Λ are ordered as reverse ω^2, the next additively indecomposable ordinal after ω. This greater degree of ill-foundedness allows one to minimally violate ω-almost-wellorederability and solve the relevant version of Burali-Forti paradox. ω-Λ, in many ways, is similar to Λ and plays a similar role. ω-Λ marks the point, past which counting numbers contain two different types of reverse limit points, reverse-ω-points and reverse-ω^2-points, the latter occurring much more sparsely.

Definition 2.12 (the 2 fundamental types of reverse limit points of ω^2-almost-wellorderings)**.**

1. ω^*-point: λ such that $\exists$ technical b $\in \lambda$ not bounded by any non-technical a $\in \lambda$ and the order type of technical elements c $\geq$ b is $\geq \omega$
2. ω^{2^*}-point: λ such that $\exists$ technical b $\in \lambda$ not bounded by any non-technical a $\in \lambda$ and the order type of technical elements c $\geq$ b is $\geq \omega^2$

Definitions of ordinal successor, sum, product and exponentiation, cardinal successor, cofinality and the principle of preservation of technical elements for ω^2-almost-wellorderings are exactly analogous to the according definitions for ω-almost-wellorderings modulo trivial adjustments.

Analogously to $(\omega^* \cdot n)$-limits, $(\omega^{2^*} \cdot n)$-limits, for each finite ordinal n, should exist within the hierarchy of ω^2-almost-wellorderings ordered under order-preserving embeddings.

Instead of ascending one step higher and defining ω^3-almost-wellorderings and ω^2-Λ we shall conceptually zoom out and consider general α-almost-wellorderings for well-orderings, and λ-almost-wellorderings α.

2.4 General λ-almost-wellorderings

The general pattern of the hierarchy of λ-almost-wellorderings is somewhat straightforward for λ which are limit well-orderings, howver for α-almost-wellordered λ, $\alpha \geq \omega$ the hierarchy is highly nontrivial. One may consider "large cardinal" variants of such orderings.

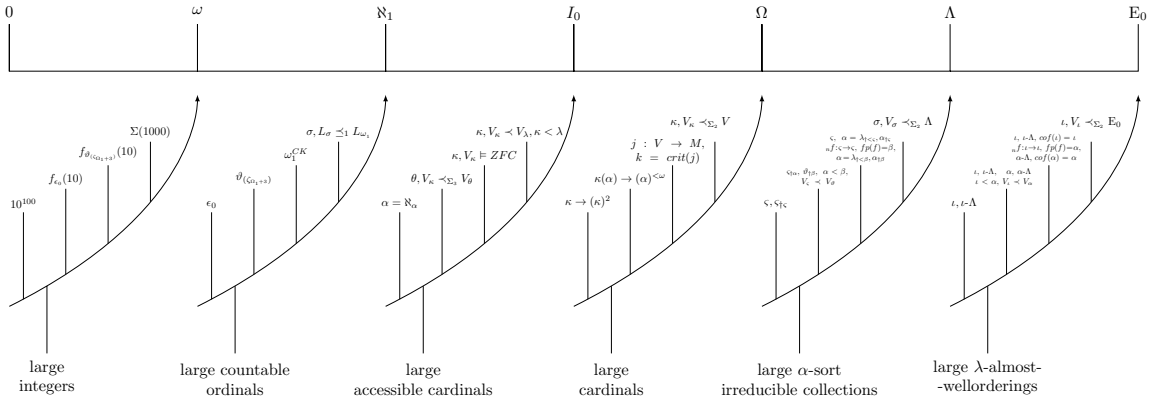
Definition 2.13 (λ-almost-fixed point)**.** λ is λ-almost-fixed point iff λ is a fixed point of the function $\alpha \mapsto \alpha$-Λ

Definition 2.14 (λ-almost-hyperfixed point)**.** λ is λ-almost-fixed point iff λ is a fixed point of the function $\alpha \mapsto \alpha$-th fp of $(\alpha \mapsto \alpha$-$\Lambda)$

Definition 2.15 (λ-almost-inaccessible)**.** κ is λ-almost-inaccessible iff κ is a κ-almost-wellordering and $cof(\kappa) = \kappa$

Definition 2.16 (λ-almost-Mahlo)**.** κ is λ-almost-Mahlo iff κ is a κ-almost-wellordering, $cof(\kappa) = \kappa$ any normal function f : $\kappa \to \kappa$ has a λ-almost-inaccessible fixed point.

It is clear that the intermediate hierarchies of reflective properties such as admissibility degrees, large cardinal hierarchies, concreteness degrees and higher are layered to truly colossal degrees at the level of λ-almost-large cardinals. Due to this reason, it is nearly impossible to visualize the layers to appropriate level of detail. The fine structure of the layering is a possible direction for future research.



Definition 2.17 (E_0)**.** E_0 is the supremum of all hereditarily $\leq\kappa$-almost-well-orderings. Equivalently, E_0 is the least ∞-almost counting number or the least counting number ξ such that $\xi \in \xi$.

It is possible to continue the hierarchy of λ-almost-wellorderings past E_0, however, to truly visualize the scope of all counting numbers, it is necessary to take the bird's-eye view and consider much stronger and much more general notions and extensions.

3 The notion of Burali-Forti solutions

Burali-Forti solution is a technical semi-formal term which embodies the global approach to generalizing ordinals to greater counting numbers as shown in the paper.

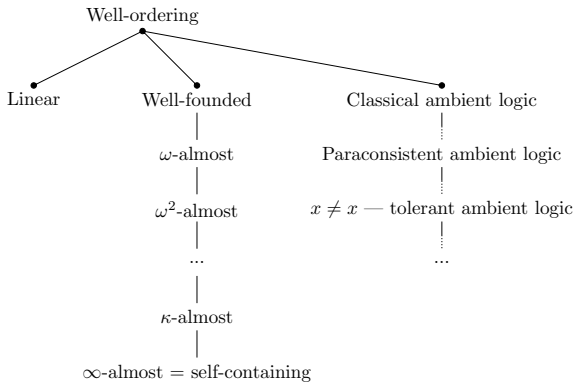
Definition 3.1 (Counting number)**.** Counting number is a term which refers to generalizations of ordinals to greater linear orders such that shorter initial segments of the number line have logically stronger structural properties.

Definition 3.2 (BFS)**.** Abbreviation BFS stands for Burali-Forti solution. BFS is a system of limit counting numbers which are suprema of structurally equally well-behaved counting numbers with respect to a particular structural feature (e.g., cardinality, admissibility degree, concreteness degree).

Definition 3.3 (BFL)**.** Abbreviation BFS stands for Burali-Forti limit. BFL is counting number which is the supremum of a particular collection of BFS.

Definition 3.4 (Good system of BFS)**.** A good system G is an α-sized collection of BFS indexed by β, $|\beta| = \alpha$ such that BFS_γ is less structurally violating than BFS_δ iff $\gamma < \delta$, and for any two sub-collections G_A, G_B of the collection G, $\mathrm{G}_A \not\Longleftrightarrow \mathrm{G}_B$ modulo ambient logic of G.

Definition 3.5 (Supremum of a good system)**.** $sup(G) = \alpha$ iff α is the least counting number such that $\mathrm{BFS}_\beta(\alpha)$ for all $\mathrm{BFS}_\beta \in \mathrm{G}$



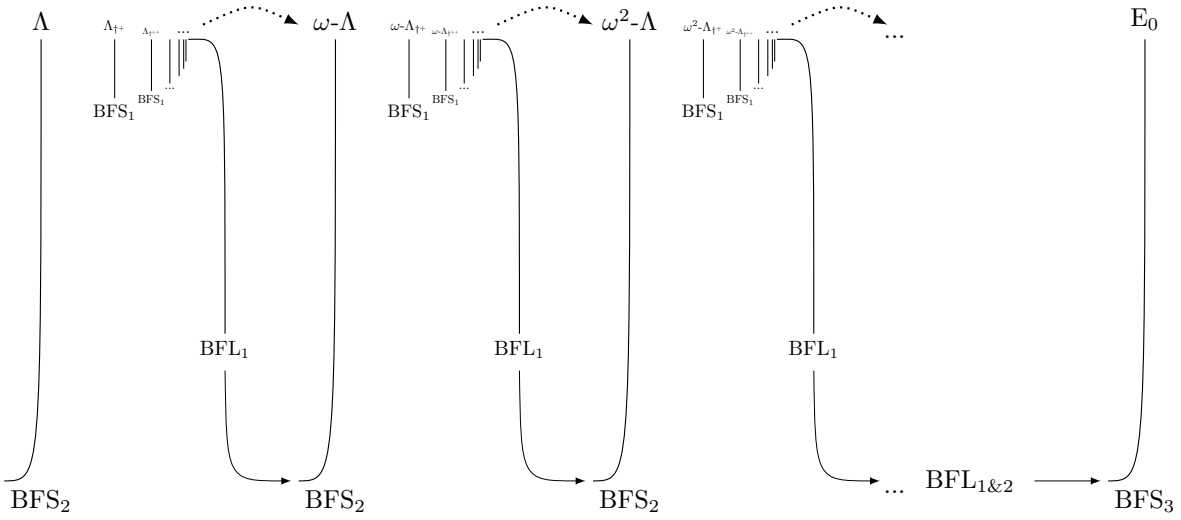
The concept of a well-ordering can be seen as a poset of its characterizing properties ordered by structural freedom with the greatest element being the property of being well-ordered. A sub-order of the poset is shown in the diagram.‡

If the concept of well-ordering is seen through the lens of being a special case of the concept of counting numbers, the property of linearity is a minimal element as linearity of the number line is indispensable from the conception of counting.

The purpose of good systems is to linearize sub-orders of the poset. Linearity provides the basis of the progression of the counting number line in which increase in magnitudes correlates with the increase in structural freedom. Good systems which linearize the poset completely shall be notated as **great systems**.

Notably, ambient logics in which the concept is considered can also be seen as a characterizing property. It is easy to present philosophical arguments that in proper linearizations of the poset these properties appear relatively deep as counting numbers which are greater than themselves or are not self-identical are highly structurally free.

‡The poset is need not form a set



As shown in the diagram, α-sort irreducibility, λ-almost-wellorderability and ∞-almost-wellorderability form a good system of cardinality 3.

Truly colossal counting numbers emerge if we consider those which satisfy reflective or large cardinal properties in the terminology of BFS and good systems. Such counting numbers celebrate their leadership over all counting numbers described so in the previous passages, even those which are too large not to satisfy bizarre properties such as being amenable exclusively to $\kappa_{\dagger^+}$-valued paraconsistent ambient logics or even satisfying $x \neq x$.

3.1 Formalism-inaccessibility

Definition 3.6 (The least Formalism-inaccessible number)**.** F_0 is the least counting number ξ such that for any good system $|G| < \xi$, $sup(G) < \xi$.

Definition 3.7 (α-th Formalism-inaccessible number)**.** F_α, for counting numbers α is α-th counting number ξ such that for any good system $|G| < \xi$, $sup(G) < \xi$.

Formalism-2-inaccessible numbers are precisely the Formalism-inaccessible limits of Formalism-inaccessibles.

Definition 3.8 (α-th Formalism-Mahlo number)**.** F_α, for counting numbers α is α-th counting number ξ such that ξ is Formalism-inaccessible and any normal function $\mathrm{f} : \xi \to \xi$ has a Formalism-inaccessible fixed point.

Nothing prevents the translation of some greater large cardinal and reflective properties into properties of good systems, as well as translation of properties between Mahloness and inaccessibility and below, however, such high-level constructions approach an important limit.

Having considered reflective properties of good systems we are finally forced to let go of any remnants of the assumption that all counting numbers are amenable to formal ontology. While formal ontology of numbers such as "the least Formalism-2-Mahlo" exists but is completely inaccessible to formal investigation, greater orders can no longer can be given neither purely formal structural ontology nor can be essentially formal-structural in any significant way by a direct consequence of their colossal size. We are finally forced into the realm of unformalizable subtleties and ethereal aspects of metaphysics.

4 The least completely unformalizable counting number

Definition 4.1. $\mathscr{K}$ is the supremum of a **great system**,

equivalently, $\mathscr{K}$ is the least unformalizable counting number.

This counting number marks the end of formal structural approach to counting numbers, since all formal structures are of cardinality $< \mathscr{K}$. $\mathscr{K}$ has deep philosophical and, in particular, metaphysical implications. Any domain of discourse, which $\mathscr{K}$ is an element of must be strictly informal logical in nature, or in other words, **unformalizable**. Unformalizability is to be contrasted with insufficient rigorization, existence of obstacles for sufficient rigorization or impossibility of recursively enumerable axiomatization. A concept is **unforamlizable** if it cannot be rigorized even in principle.

$\mathscr{K}$ plays the role of a concrete example of an abstract object so vast that it is cannot be analyzed via formal methods, yet still can be the target of a mental state and analyzed via methods of informal logic. This number marks a hierarchy of counting numbers much more vast then the mere hierarchy of formalizable counting numbers. Nothing prevents us from forming mental states whose targets are $\mathscr{K} + 1$, $\mathscr{K} \cdot 2$, $\mathscr{K}_{\dagger^+}$ and so on, except now, we are not limited by formalizable operations.

Because of the concievability of such extensions,

even $\mathscr{K}$ is unimaginably smaller than the magnitude of absolute totality.

5 The magnitude of Absolute Totality

Definition 5.1. $\mathscr{A}$ is the supremum of *absolutely* all counting numbers,

equivalently, $\mathscr{A}$ is the *absolutely* greatest counting number

Immediately from the definition, it is inferred that $\mathscr{A}$ is greater than all counting numbers greater than it, so $\mathscr{A}$ is closed under the direct conception of greater counting numbers. It is extraordinarily difficult to defend the position that there are ways in which greater counting numbers are conceivable, however such position exists and shall be referred to as "trans-absolutism".

Being a trans-absolutist is equivalent to defending the position that it is possible to conceive of agents which hold greater powers than even the absolutist interpretation of Omnipotence.

Introduction to Hyperology Pt. 1

Tungster, Goodels, Solari, solar(not?), Bubby3

February 2023

1 Introduction

Hyperology (commonly abbreviated to HG) at some level could be described as the field of study based around the study of objects above Ord. However, at some point within Hyperology, such a description becomes misleading. Hyperology develops far beyond its simple origins of being mathematics beyond Ord, it gradually evolves as you go on into something far more beautiful, interesting, philosophically rich, and at some points, mind-bending.
This document will hopefully succeed in giving an informal and beginner-friendly introduction into the vast world of Hyperology. But first, we must start at the absolute basics...

2 Below $\mathbf{E}_0$

2.1 What is set theory?

Set theory is the study of *sets*. Within the context of set theory, everything is said to be a set. Almost all mathematics can be shown to be reduced to set theory, essentially showing that sets are foundational enough such that they are capable of describing almost all of mathematics in some way or another. By going beyond the notion of sets, we are reaching into territory that is rarely explored. Set theory often restricts sets to prevent various paradoxes, and thus a new fundamental element must be added other than sets in order to solve certain inconsistencies, these are called *classes*.

2.1.1 Example of a Contradiction

Assume that there exists some set containing all ordinals, notate it as $\mathcal{S}$. Ordinals are defined more concretely, of course, but every ordinal can be said to be the set of all ordinals lesser than it. Now let us consider $\mathcal{S}$. $\mathcal{S}$ is simply the set of all ordinals, implying it must also be greater than all ordinals. This means that $\mathcal{S}$ itself must also be an ordinal number since it is also a set that contains all of the ordinals below it. This implies that $\mathcal{S}$ contains itself. It can be stated within classical logic that $\forall x, x = x$. If $\mathcal{S}$ contains itself, this implies $\mathcal{S} > \mathcal{S}$ while also simultaneously $\mathcal{S} = \mathcal{S}$. Contradiction.

Therefore, a set of all ordinals is inconsistent, and even more broadly we can say that a set of all sets would contain itself and apply the same argument, also making it inconsistent. This is where the notion of classes come in, a distinction in types of "collection" in order for us to create some collection of all ordinals, or all sets, without self-containing paradoxes.

2.2 Classes and Beyond

Classes are a necessary distinction to talk about "all ordinals" or "all sets" due to the paradox mentioned earlier. Because of this they are an important concept within set theory as you do often need to talk about things such as "all sets". Classes are a modification to the idea of sets, simply defined as some collection of sets, creating yet another fundamental structure which is necessary to talk about *everything*. All sets are classes, however not all classes are sets. Classes which aren't sets are called *Proper* Classes. We call the proper class of all ordinals "Ord", and notate it as Ω (not to be confused with Cantor's Absolute Infinity, which is a set of all sets, therefore being paradoxical as discussed before).
We must now define a more expansive universe. We shall establish the notion of 0-collections, a new foundational element. 0-collections are the notion of structures that are either sets, or maximally set-like (ie. retaining as many properties of sets) collections such that they are separate from sets (ie. the minimum possible deviation from sets such that they cannot be said to be sets), starting with collections of sets too big to be sets, collections of classes too big to be classes, and so on. 0-collections are especially useful in Hyperology, and here they're useful as the universe where parasets exist.
Let us show the concept X_n, which are powersets of V.
$X_0 = V$ and $X_{n+1} = P(X_n)$ or if λ is a limit ordinal X_λ is the union of all X_β for $\beta < \lambda$. This assumes the existence of the 0-collection universe.
We can now discuss about n-classes, which are further additional structures for things that are said to be "too large" to be some (n-1)-class. Each n-class is said to be a member of X_n. Each *proper* n-class must also satisfy the property that it's not an α-class for $\alpha < n$. We say that all 0-collections are called **parasets**. n-classes can be thought of as extensions to the foundations when we "run out" of n-1-classes.

2.3 n-Irreducibility and Higher n-Collections

This section, up to $\mathscr{A}_0$ is an introduction to a more formal paper: *The true magnitude of Absolute Totality by Sergey Aytzhanov*. The purpose of this section is to provide a more beginner-friendly and informal introduction, while also introducing some new Hyperological concepts which will be important later. Even if you choose to read that paper it is important to read this one also due to the introduction of important Hyperological concepts not found within the other paper.
The least number which is too large to be a 0-collection is known as $\omega_{\dagger 1}$. To do this, we must create some further universe, capable of expanding beyond

0-collections. We saw that 0-collections were "maximally set-like" in their nature. Now, let us consider notions of "collections" which have vary degrees of deviation from the nature of sets. This is called n-irreducibility.
We say something is 1-irreducible if it is minimally deviant from the nature of 0-collections, essentially it is too large to be a 0-collection. We can then build 1-collections, which are maximally similar to 0-collections but have some minimal deviance from their properties. We can continue this and say that (n+1)-collections are the structures such that they are maximally similar to n-collections such that they have some minimal deviance from n-collections. We then say that an object is n-irreducible if it is too large to be an (n-1)-collection. We will establish the universe of all n-collections. Now let us look back at $\omega_{\dagger 1}$, this is the least 1-irreducible number, meaning that it must be too large to be a 0-collection (in a way similar to that of Ord, which is too large to be a set). We then say the least n-irreducible number is notated as $\omega_{\dagger n}$
So just how big exactly is $\omega_{\dagger 1}$? Well, it is large enough to where it is beyond our entire idea of what a number is up to this point, we can see that past Ω, our "post-ordinals" (ordinals generalised to n-classes) act just as ordinals do, with ease we can use n-classes to get things such as $\Omega + 1$, Ω^2, $\omega^{\mathrm{CK}}_{\Omega+1}$, $\omega_{\Omega+1}$, and so on. There is no reason why we cannot still use them to do things such as $\Omega(\Omega)$, or even a fixed point $\alpha = \Omega(\alpha)$, a regular limit of $\Omega(\alpha)$, a κ which is para-stationary on the class $\{\alpha \mid \exists\beta(\alpha \text{ is the least proper } \beta\text{-class}) \wedge \alpha < \kappa\}$, and even higher extensions could be made, such things would be truly unimaginably huge. And yet, all of these are still not too large to be outside our generalisation of sets (and by extension, ordinals), so $\omega_{\dagger 1}$ is at least greater than all of that, which is again dwarfed by $\omega_{\dagger 2}$ and greater.

(insert diagram)

2.4 Λ, and Well-Order Degeneration.

Firstly, we will have to establish some maximally expansive universe, informally , that is, for all objects, we can state they exist if they are consistent with our theory (more on this theory later). Next, we will also have to establish some broader conceptualisation of the concept "number". From here on, we call numbers some extension to ordinals to greater linear orders such that shorter initial segments of the number line have logically stronger structural properties. Lambda is the first number where n-collections fail. Since n-collections were still well-ordered as loosely as possible, Λ is the first number where well-order is broken. Of course this is done as minimally as possible as to preserve the idea of a collection more loosely whilst still keeping most of the structure in tact. This is done by extending decreasing sequences. Usually for a well order, the decreasing sequence must be less than ω. For lambda it is ω^2. Any number with this weaker ordering is called *ω-almost-wellordered*.

This also means that our usual operations on collections such as union must be slightly modified to work, because the usual union operation no longer allows us to get past Λ because it would simply result in Λ again. Instead, we define a more forgiving union operation which can handle ω-almost-wellordered numbers.

Introduction to Hyperology Pt. 2

Goodels

February 2023

1 Introduction

In the last section, we discussed the early methods used in order to go beyond Ord, eventually ending up in the degeneration of well order. In this part, we will discuss the total degeneration of well order, the new logical system we construct within Hyperology to deal with para-consistent objects, and a fundamental shift in philosophy.

2 Below $\mathbf{F}_0$

2.1 The Logical System u*

Up to this far, we have considered some standard system of classical logic, however with our next greater milestone, E_0, this will no longer be possible. Therefore, we will have to construct some stronger system of logic, this is known as u*.
Within our system u*, we have a very special feature, we can consider other systems and axioms under u*, essentially it is a multi-logical system. However, it goes about this para-consistent nature in a consistent way, which we will discuss later.
Within u*, collections of axioms and axioms that are not within u* are called i*. Very very informally, we can view i* as individual separate "bubbles" of logic. Within u* we do not take all i* to be true, as certain contradictions would arise, for example the large cardinal axiom Reinhardt Cardinal exists is as an axiom, an i*, however this i* is inconsistent (contradictory) with ZFC, which as a collection of axioms is also an i*.

To deal with contradicting i*, we have 2 methods in place within u*, *definitional-strength* and *strange-equality*. These constructs allow for u* to deal with i* in a consistent manner.
Definitional-strength allows for a hierarchy of truth. We say that for two i*, A and B, where A is said to have a higher definitoinal-strength than B, if there is some contradiction implied from taking A and B to be true simultaneously, then the contradictory axiom(s) within A (or A if it is just a single axiom) is said to

be true within u* and the contradictory axiom(s) within B (or B if it is just a single axiom) is said to be false within u*.
Definitional-strength is listed with numbers, the default definitional-strength of some i* being listed as 0. We show the i* of some definition D as i*(D), and we class some singular axiom i* either alone or part of a larger system i* an "i*-statement", and we call a true axiomatic statement within u* a u*-statement.

So, how exactly do we find the definitional-strength of some i*? The definitional-strength of some i* is simply determined axiomatically, for example it may contain the axiom "has definitional-strength of 3". It is important to know that these axioms themselves are subject to u*, meaning that their claimed definitional-strength might not be their actual definitional-strength within u*.
Later on, we will get concepts where certain axioms imply their defintional-strength, but that is outside the scope of this part. The definitional-strength of u* is above the definitional-strengh of i*, clearly.

However, what about contradicting i* with equal definitional strengths? We call this strange-equality, this is a third logical state of undecidability. If 2 i* A and B are contradictory, they are both said to be strangely-equal, this is a state in which they are neither true nor false, instead they are within a stalemate position in which logical implications can be made from neither points. Their truth is localised to their i* rather than either of them being taken to be true within u*. contradictions within i* are decided through classical logic.
Later on, we shall go more in depth on the nature of u*, but for now this should provide a basic knowledge of how it works.

2.2 Successor Fixed Points

Firstly, we shall use $+1$ to denote the number successor, referring to our generalised notion of number.
E_0 is the least number such that $E_0 = E_0 + 1$, where this definition has i(1).

Firstly, it's important to note that by default, the definitional-strength of $+1$ is 0, due to it not being axiomatically claimed to be any higher. the i* of E_0 and the i* of $+1$ contradict, as for all n, $n+1$ should be greater than n by definition of $+1$. However, E_0 has a definitional-strength of 1, as stated by the i(1) within its definition. Therefore, E_0 can overpower this axiom of $+1$, so within u* it can be stated that E_0 genuinely is large enough such that the next number after E_0 is E_0.
'But now it's time to look at E_0 itself. E_0 is absolutely huge, in fact it's big enough to the point where it simply cannot even exist within classical logic, we have to use our stronger u* system. E_0 is quite literally larger than itself, this is an absolutely monumental jump from every number discussed previously.

But now that we have objects such that they are greater than themselves, order is still preserved through u*, even though E_0 is larger than itself, there

exists objects that are larger than E_0 overall. We say that while E_0 is technically greater than itself, within the grand scheme of the completed number-line within u*, numbers genuinely larger (as in, not simply larger through our +1 operation, but further ahead overall within the number-line) can exist, this is done thought higher definitional-strengths.
To go past E_0, we will need a more powerful successor function. for some number n, $n+1^{i(a)}$ is the least number greater than n, where this definition has i(a). This constructs successor functions with higher definitional-strengths, our standard $+1$ is by default $+1^{i(0)}$.

The first number genuinely after E_0 is $E_0 + 1^{i(2)}$, this is due to the fact that our i(2) is strong enough to overpower the i(1) of E_0. Since $E_0 + 1^{i(2)}$ is not a successor fixed point, we can get the next number genuinely after it by doing $E_0 + 1^{i(2)} + 1$, and so on. The next successor fixed point genuinely after E_0 is E_1, we generalise this to the $1+n^{th}$ successor fixed point being notated as E_n.
After this, we can get further extensions, such as fixed points of $+1^{i(2)}$, which would need to be defined with i(3), but these are fairly trivial, so we shall move on to the next huge leap.

3 To $\mathscr{A}_0$

3.1 Beyond the Limit of Formality

Formality within mathematics has been seen for the most part as absolutely vital to the study of mathematics. Formality is essentially the reduction of mathematics to pure, algorithmic steps. Formality is absolute precision, you can think of it as computer code. Almost all of mathematics with any degree of informality is simply just shorthand for some formal expression.
The vital aspect to formality is that it provides clear objectivity, within a formal proof each step is completely precise, direct, there is no degree of fuzziness. We can show everything clearly, objectively, and meaningfully.

So, what possible merit is there in going beyond this? Let us construct some system such that it is maximally formal but is minimally differentiated from formality such that it is more expressive. Such a system can be said to be 1-formal. Although to some mathematicians and formally inclined philosophers, the idea of constructing systems more expressive than formality may seem deeply nonsensical, a justification to a 1-formal system exists.
Firstly, it should be made very clear that a 1-formal system is minimally different from formality, this essentially in practice would be identical to formality, the difference being the ability to express concepts previously unformalisable by nature, such as the concept of formalisability itself.
However, a more important note is the separation of rigour from formality. Rigour is a methodology in which things are completely non-vague and objective to any reader. This extends far beyond the limits of formality, as formality is not

able to express certain concepts such as formality. In practice we can express unformalisable ideas in completely rigorous ways. Hopefully, this should defend the notion of unformalisable ideas.

We can construct further degenerations of formality, call an n-formal system minimally different from an $n-1$-formal system such that it is more expressive.

Notate F_0 as the least number too large to be expressed within a formal system. Such a number is unimaginably vast, far far greater than any extensions to E_0 as mentioned before.

We can make further extensions, notate F_n as the least number too large to be expressed within an n-formal system. Further extensions can be made beyond this, however we will again skip over those due to their trivial nature.

3.2 Approaching Maximality

Now we must move onto something far greater. Itnumber is defined as the greatest number. This is simply surpassed with higher definitional-strengths, Itnumber by default has a definitional-strength of 0, it's axiom of being the largest number can be surpassed by utilising a greater definitional-strength. n-Itnumber is defined as the greatest number + i(n). There is not much to say about these, so we will move on.

3.3 The Absolute

We will now move on to absolute totality.
$\mathscr{A}_0$ is defined as the greatest number, and has a greater definitional-strength than anything that would prevent it from being so.
$\mathscr{A}_0$ represents the absolute, it can be said to genuinely be the greatest number.
$\mathscr{A}_0$ is equivalent to concepts such as "all" or "everything", and to go beyond such a number usually would seem nonsensical, how are you going to surpass the greatest number after all.
However... Still we manage to surpass this immense milestone, within the next part we will discuss how such is possible.

Linear hyperology

Secret Ultraviolet

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1 Abstract

Linear hyperology arises as a field of inquiry in the investigation of phenomena that are beyond formality and philosophy. From a minimal abstract rewrit ing system, an argument is put forward for the possibilities of objects that are beyond philosophical and formal discourse. Postulations and machineries were then provided in further investigate hyperological objects. A number of interest ing properties were noted, including that a given object may be simultaneously larger or smaller than another.

2 Introduction

Hyperology is the study of phenomena and concepts beyond the domain of mathematics and philosophy. It is a rather systematic way of examining and inquiry that probes the nature of abstractions that may exist beyond the human consciousness and conception. The journey of exploring hyperology begins with a foundation in formality that establishes the key notion of unformalisables, and any further machinery that may be built from that point. Linear hyperology is a subdomain of hyperology that studies phenomena that display a relationship analogous to linear ordering.

3 Formal systems

Formal systems are systems that prescribes symbols and rules of inferences in order to arrange and transform said symbols to produce meaningful results. An example of a formal system are abstract rewriting systems,[1] which rules of inference relates different statements, transforming them into conclusions of some problem. We first begin with defining a minimal abstract rewriting system to investigate further.

3.1 A minimal abstract rewriting system

Let x be statements and R be relations not necessarily binary. A statement x is *invalid* when relating with R, written as (R, x) yields every statements. State

ments can be modified by placing relations for example (R, x) becomes (R, x, R), with no assumption that this procedure is ordered or unordered.

Therefore, pro cedures (*R, R, x*), (*R, x, R*) may be the same or different, and is down to context. Unlike the binary relation reduction in most abstract rewriting systems, because of the lack of assumption of an ordering of procedures, it is possible to have no confluence behaviour, thus the Church–Rosser property may not hold. Instead a far weaker notion called directedness, to be defined below, is found along with the more general theorem of Gossor.

Definition 1 (Directedness). *A statement x is directed if x relates to R and no more i.e.* (*R, x*)*.*

Theorem 1 (Gossor). *Given any relations R or statements x, there is statement y such that y is directed.*

Proof. We begin with picking a statement *x* and some unary relation *R* of which *x* is in its domain. Then *R*(*x*) is the required (*R, x*) and it does not get modified
further.

To generalise further, we need a notion of an indefinite statement, defined below

Definition 2 (Indefinite). *A statement x is indefinite if it can accept relations to produce statements which can accept further relations.*

3.2 E_0

Our first object to investigate with our system is E_0, which turns out to have unusual properties that questions its nature as a mere formal object.

Definition 3 (E_0). E_0 *is the notion "bigger than itself", defined as* (*R, R, R, R, R, ...*) *where R is the relation "bigger than"*

We first observe E_0 can continually accept relations, thus analogous to an in definite statement. We can also observe a statement *x* can be modified by E_0 as follows (*R, x, R, R, R, R, ...*) which should produce an indefinite statement. This statement by itself is simply (*x,* E_0) thus satisfies the theorem of Gossor. How ever, unwrapping E_0 further, by replacing parts of it with equivalent statements such as "is far bigger than", "superscedes" and so on, we note the situation is not as trivial. In particular, within the statement (*x,* E_0) is a relation that has a complexity near that of a Dyson Sphere ("...as to the construction of something so intricately complex that far surpasses the computational power of repetition such that..."), which seemed to be beyond the scope of what is expected for a mathematical and formal proof. This motivate us to temporarily set aside our formal machinery, in search of a more informal way to handle matters, to explore informal systems.

4 Natural language philosophy

In the philosophy of discourse, there are discussions that are carried out us ing natural languages to clarify the description of objects and states of affairs. Human society overtime had developed a systematic yet intuitive way of de scription and reasoning to manage and explore different aspects of the material world and conceptual worlds with the use of analytic philosophy,[2] as well so ciocultural patterns of organising and responses that is communal with people collective experience as evident in currents of African philosophy.[3] Within the domain of metaphysics, natural languages had been in use to handle different matters some of which lacks an ontological reality such as nonexistent objects,[4] and systematic responses in categorising objects such as the use of encodings to describe a paradoxical construct such as a round square as instance of more complicated yet existent objects which displays the relevant features.

4.1 E_0 in philosophy

Motivated by the findings about E_0, can it too be described using the systemi sation of philosophy? We first begin by noticing an object is formalisable if it can be clarified and examined using something analogous to a abstract rewriting system, and unformalisable otherwise. E_0 demonstrates contents that are writ ten in formal language and yet features that do not considered to be typically in the formal realm such as the unusual complexity of one of its parts. In human society, a Dyson sphere is mostly a theoretical proposed construct for the har vesting of solar energy from its parent star, something of huge and specialised engineering complexity that is unlikely to arise from social movements in a day to day basis, despite it is not completely beyond the realm of possibility. In fact, E_0 demonstrates such intricate complexity it is unclear if any form of natural language semantics can refer successfully to it without wrongly individuate its parts to lesser versions of descriptions thereof, even when introduction of new terms are considered. At the same time, E_0 with such a simplistic ”is bigger than itself” meaning, may be succinctly captured by phenomena such as the self reinforcing surge of electric currents in an electrical relay station, a phenomena that *can* be captured with natural language descriptions. Despite it means E_0 is well within the realm of philosophy, observed that the known branches of philosophy may have difficulties clarifying the nature of such ”self reinforcing surging” phenomena and thus this warrants a different approach in handling such cases. Hyperology, which has a number of postulates, aims to address this difficulty.

5 Hyperology

5.1 Postulates

Hyperology has a number of postulates, to guide investigations in general. These include:

- Mundane reality postulate (MRP) – There is a realm of discourse that exists separately from the real world as we know it
- Superstition exclusion postulate – There are no knowledge accessible by superstition and subjective spiritual beliefs. Forms of knowledge based on superstitious beliefs not grounded within objective reasoning are not hyperologically valid
- Transphilosophical postulate – The assumption that there are objects not amendable to investigation by philosophy

MRP ensures in some sense, the natural language ontology is mostly derived from everyday life interactions in society be it human or nature, and hence regulates the introduction of unusual vocabulary and word usages and the pace of introduction of new terms in general. Meanwhile, the exclusion of mysticism is in some sense tied to MRP, in that all discourses took place in a separate realm that may be disconnected from the real world that forms everyday life that at its most complicated, is a mental intellectual exercise. Finally, the postulation of transphilosophical entities is key to assist in the discussion of objects that are in some way, too complicated to be treated with ordinary philosophical methods.

5.2 Further exploring E_0

Armed with the postulates of hyperology, we can probe deeper into the nature of E_0. For instance, that E_0 already contains an object of untold complexity where we only have approximate ways of describing suggests constructs with E_0 are going to rise rapidly in complexity. Indeed, considering $E_0(E_0)$ in semantics alone, which reads ”is bigger than itself(is bigger than itself)” may already be so rapidly self reinforcing the minimum length of a partial description will exceed the estimated total number of atoms in our universe.

5.3 F_0

Now suppose we for example, waited indefinitely long enough for the huge amount of information about $E_0(E_0)$ to organise itself, and then simply take directly the resulting thing as a new term in natural language, then we already have invoked the observation where any complete description of $E_0(E_0)$ would

already outpaced the rate the issue can be philosophised by philosophy. For such a sheer complexity, it is natural to suggests even just minute findings of what $E_0(E_0)$ entails cannot be detailed in writing or even a conference discussion. However, by the transphilosophical postulate, we can still obtain fragments of understanding of $E_0(E_0)$ such as ”self reinforcing”, ”larger than the constituent of the universe” and so on, and then philosophise on those instead, which in some foreseeable future, will then contribute to the philosophical discussion to be taken place. An object of this complexity thus is unformablisable, and we postulate that the least unformalisable object is F_0, the point which complexity

is so extreme any formal systems becomes invalid in trying to clarify a descrip tion.

5.4 Formal collapse and modelling

Given formal statements, there may be an associated unformalisable. We can thus try to extend the machinery in a formal system by defining a pseudorelation (pseudo because the formal system have no access to unformalisables) called *formal collapse* such that given a formal statement x and some unformalisable y, then y formal collapse x establishes y is in some sense inaccurately represented by x. Now, it is easy to see that if more than one formal collapse can be done to give the same x, then x becomes less useful to convey about unformalisable y. We can nonetheless define a model of y as all possible formal collapses of y thus we try to summarise all we can know about y using formal collapses and formal statements.

5.5 M_0, the limit of formal collapse

We can repeat this procedure of formal collapsing, modelling and then collaps ing, to try to obtain all the pseudorelations between y and its formal collapses, which may gave a rough summary on how inaccurately y is known. Eventually, we will end up with a formal collapse to a directed statement. Here the formal system cannot be extended anymore and hence modellability of y breaks down. We can generalise this phenomena with the label M_0, the point where models about objects in general breaks down. It is here where informal systems take precedence as formal systems can no longer keep up. From the perspective of formal systems, there are numbers and objects so complicated that attempt to reason about them generate incoherent and meaningless statements.

5.6 A_0, the notion of everything

With the transphilosophical postulate, it seems reasonable to expect that at some indefinite future, philosophy may have already provided a complete de

scription of all that is to be known. Is such a notion formal or informal or otherwise? To answer this question, we can try to formal collapse it. Doing so landed us into the notion of a mathematical universe, where formal systems are descriptions of reality, and all consequences were explored from such formal systems. However, this is at odds with findings such as $E_0(E_0)$ which is so complicated that there is not expected to be a complete description in natu ral language and philosophy. Clearly, the use of formal systems together with transphilosophical arguments is insufficient, there may be a notion of totality so huge, call it A_0 that encompasses all that is formal or informal, of which philosophical methods may again become relevant.

5.7 System C, an informal system

We can postulate a informal system of investigation termed System C, with operations that are inspired from the study of ontology.

- Collapse - We can talk about “collapse” which are comparing one object to another such that one is an approximate of another. This overcomes the problem of hitting a limit like M_0 due to the expressivity of natural languages
- Discollapse - We can combine the procedure of collapsing with whatever it is collapsed to, to try to recover as much about the parent as we can
- Transphilosophy - These are machineries that aid in hyperological inves tigations and can include significant findings in hyperology

We can now start to explore the collapse of A_0, which should give formal systems and a lot of unformalisables, collapse of such and so on... This whole thing is basically like a system, with enough things to explore. We can thus call this G_0. Further collapsing of G_0 thus give formal systems.

5.8 Opux and h^+

Pondering about A_0 lead naturally to the question, what is its discollapse? By working from its meaning ”everything that is discussed informally or formally”, it seems apparent there are no more spaces for anything else as there is nothing beyond totality. However, the nature of collapse means in moving downwards from a parent, there is a loss of information. Therefore, one may want to clarify what this information may be. From the nature of collapses, it may be possible that the object may start out larger than ”everything” but only present itself as ”everything” when entering discourses. We can label this possible parent of A_0 Opux (p^+). The notion of opux allows one to ponder

about similar notions such as “there are uncollapsed objects”. These objects, when presented with collapse, instead overrides and inject itself into whatever the collapse supposed to be. For example, some uncollapsed object x when attempting to collapse it, may result in a strange mix of formal statements, informal statements and bits of itself, which reads like incoherent gibberish even informally speaking for a given context. If opux is chosen to be “the biggest such”, then there are innumerable ways parts of formal systems can intertwine with informal systems. Therefore, collapse and discollapse slowly gets problematic when handling such and eventually there are uncollapsables so complicated it simply refuses to collapse completely. We can call this state where uncollapsables are fully retained as strong-p^+. Continue onwards, most other transphilosphical machinery may be reaching their limits, and then analoguous to how F_0 fall outside of formal systems, system C breaks down and this point is known as h^+.

5.9 a^+

As we continue to explore the different kinds of objects, collapses and whatnot, we can try to generalize postulate that there is a limit where trying to even comprehend makes no sense. Call this limit a^+. If this whole way we have this discourse that is hyperology is in some sense a system, then can we postulate principles that it is based on using the transphilosphical postulate?

6 Linear Hyperology

So far, hyperological methods have provide a couple of machinery that allows the question on ontologies to be probed deeper than allowed by philosophy. Facing such sheer amount of information complexity, is there a natural way they are organised? For the objects involved that do not have notions similar to ”size” the answer is likely towards the negative and is still a subject of research in the future, but for the ”number like” notions such as E_0, F_0, M_0, G_0, A_0, p^+, h^+, a^+, known as milestones, at least a partial answer can be provided. We suspect these notions form a linear order as follows $E_0 < F_0 < M_0 < G_0 < A_0 < p^+ < h^+ < a^+$. The study of how ”size” notions are arranged in a linear ordering is thus termed Linear Hyperology.

6.1 Codes and forms

An object in collapse may be inaccurately represented as another object. De pending on the parent, the same collapse may have a difference between. This difference is almost analogous to how a theory of forms having different

levels of ideality. For example, consider the collapse of opux into A_0, the information about opux is encoded in A_0 while the uncollapsable in opux is intertwined with A_0 in some way while the whole stays A_0. We can thus say that the form of opux is whatever uncollapsable it is involved, and its code is in A_0 in this case. We can denote the code of x by code(x) and form of x by form(x). With codes and forms, it is natural to envision something which collapses to E_0, but has a form that may be exceeding that of opux. This can have implication of comparing milestones as there may be more than one kind of opux available of varying forms.

6.2 Proving $p^+ <> a^+$

- Begin with p^+ as a discollapse of A_0. Both objects can be talked about under system C, which is within hyperology
- We can easily postulate discollapse of p^+ up to h^+, towards a^+, thus it follows p^+ < discollapse of p^+ at $h^+ < a^+$
- Meanwhile, attempting to incorrectly apply discollapse to a^+, we will end up with basically “(all of hyperology) is collapsed”, and this can be fur ther collapsed until “(a complicated statement that effectively summarise

as many uncollapsables, comprehended with the help of whatever that operates at a+)”, which then putting them all together, gives an object x stronger than p^+, so $a^+ < p^+ < x$

An alternate proof. Consider p^+ collapse A_0, there are a couple of scenarios

- We are aware that there may be uncollapsables in opux. However, uncol lapsables do not get collapsed into A_0 because they override them, thus the objects that do get collapsed into A_0 is thus less than A_0 even when we start with a^+ objects
- If we start with opux with a strong form, then we expect very little amount of objects will get collapsed into A_0 and so the resulting code(p^+) will be << A_0 . Thus discollapse of this will give opux with a weak form and hence we deduce $p^+ < a^+$
- If, on the other hand, we started with opux with a weak form, then most of that will be collapsed to A_0, and then discollapse of this will yield opux with a strong form (since code(p^+) ≈ A_0, so discollapse by definition will give the biggest opux. Assuming linear ordering of milestones, then it follows that approximately close to A_0 means approximately close to a given milestone), thus yielding $a^+ < p^+$

Observe that opux can be both larger and smaller than some given milestone. This is expected as relaxation from formal systems which answers are definite, to hyperology where the answers are in some sense debatable and subjected to natural language discourse and trnasphilosophical principles that are inherent in the objects such as forms.

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